

# AN ANALYTIC STUDY OF NONSTEADY TWO-PHASE LAMINAR BOUNDARY LAYER AROUND AN AIRFOIL

by

Professor YU-KAO HSU  
Department of Mathematics  
University of Maine  
Orono, Maine 04473

Recently, NASA, FAA, and other organizations have focused their attention upon the possible effects of rain on airfoil performance. Rhode<sup>1</sup> carried out early experiments and concluded that the rain impacting the aircraft increased the drag. Bergrum<sup>2</sup> made numerical calculation for the rain effects on airfoil. He claimed that thin airfoils of different sections having the same exposed frontal areas, will have approximately the same rates of water-drop impingement at high speeds, but the distribution of water-drop impingement will be different for each section. Luers and Haines<sup>3</sup> did analytic investigation and found that heavy rain induces severe aerodynamic penalties including both momentum penalty due to the impact of the rain and a drag and lift penalty due to rain roughening of the airfoil and fuselage. More recently, Hansman and Barsotti<sup>4</sup> performed experiments and declared that performance degradation of an airfoil in heavy rain is due to the effective roughening of the surface by the water layer. Hansman and Craig<sup>5</sup> did further experimental research at low Reynolds number. They concluded that the initial effect of rain is to cause premature boundary layer transition near the leading edge.

E. Dunham<sup>6</sup> made critical review for the potential influence of rain on airfoil performance. Dunham<sup>7</sup> et al. carried out experiments for the transport type airfoil and concluded that there is a reduction of maximum lift capability with increase in drag. There is a scarcity of published literature in analytic research of two-phase boundary layer around an airfoil. Although Henry<sup>8</sup> et al. presented a technical paper entitled "A Von Karman Integral Approach to a Two-Phase Boundary Layer", yet their main assumption that the existence of zero shear stress at the wall and on the interface is not physically realistic. Most recently Bilanin<sup>9</sup> attempted an analytic investigation.

He assumed that the ejecta layer thickness is constant in his preliminary report. This assumption is quite doubtful. The present author attempts to improve the analytic research. The following assumptions are made:

1. The fluid flow is non-steady, viscous, and incompressible.
2. The airfoil is represented by a two-dimensional flat plate.
3. There is only laminary boundary layer throughout the flow region.

Under the usual boundary layer approximation, there obtains,

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$$\begin{array}{lcl} \text{For the liquid:} & \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \alpha \frac{V_j \sin \beta}{\delta} & (1) \\ \text{Eq. of continuity} & & \end{array}$$

$$\text{Eq. of momentum} \quad \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{(Re)_L} \frac{\partial^2 u}{\partial y^2} + \alpha \frac{V_j^2 \sin \beta \cos \beta}{\delta} \quad (2)$$

$$\text{Where} \quad (Re)_e = \frac{P_w V_c L}{\mu} = \text{Reynolds number of the liquid water}$$

$$\alpha = \frac{W_L}{P_w} = \text{ratio of liquid water content to water density}$$

$$\delta = 5 \sqrt{V_x / U_\infty} / L$$

For the fog:

$$\text{Equation of continuity} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$\text{Equation of momentum} \quad \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{(Re)_L} \frac{\partial^2 u}{\partial y^2} \quad (4)$$

The initial and boundary conditions are:

$$\text{at } \tau = 0 \quad u = U(x, 0) \quad V = 0$$

$$\text{for the liquid phase,} \quad \text{at } y=0, \quad u=v=0 \quad (\text{No slip condition})$$

$$\text{at the interface } y=\delta$$

$$\begin{aligned} (u)_L &= (u)_f \\ (V)_L &= (V)_f \\ \left( \mu \frac{\partial u}{\partial y} \right)_L &= \left( \mu \frac{\partial u}{\partial y} \right)_f \end{aligned}$$

$$\text{as } y \rightarrow \infty, \quad u = U(x, \tau)$$

The above set of partial differential equations is non-linear in nature. An exact solution is not possible. The said set of partial differential equations is transformed into a set of finite difference equations. Using fortran language. An numerical solution is expected.

References

## References

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